

ST BONAVENTURE KAHETI BOYS HIGH SCHOOL

FORM 2 MATHEMATICS

WEEK 4

AREA OF A TRIANGLE

Specific Objectives

By the end of the topic the learner should be able to:

- Derive the formula; Area = $\frac{1}{2}ab \sin C$
- Solve problems involving area of triangles using the formula Area = $\frac{1}{2}ab \sin C$;
- Solve problems on area of a triangle using the formula area = $\sqrt{s(s - a)(s - b)(s - c)}$

Content

- Area of triangle $A = \frac{1}{2} ab \sin C$
- Area of a triangle $A = \sqrt{s(s - a)(s - b)(s - c)}$
- Application of the above formulae in solving problems involving real life situations.

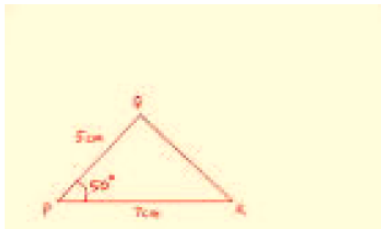
Introduction

Area of a triangle given two sides and an included Angle

The area of a triangle is given by $A = \frac{1}{2}bh$ but sometimes we use other formulas to as follows.

Example

If the length of two sides and an included angle of a triangle are given, the area of the triangle is given by $A = \frac{1}{2}absin\theta$



In the figure above PQ is 5 cm and PR is 7 cm angle QPR is 50° . Find the area of the the triangle.

Solution

Using the formulae by $A = \frac{1}{2}absin\theta$ a= 5 cm b =7 cm and $\theta = 50^\circ$

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 5 \times 7 \sin 50^\circ \\ &= 2.5 \times 7 \times 0.7660 \\ &= 13.40 \text{ cm}^2 \end{aligned}$$

Area of the triangle, given the three sides.

Example

Find the area of a triangle ABC in which AB = 5 cm, BC = 6 cm and AC = 7 cm.

Solution

When only three sides are given us the formulae

$$A = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{Hero's formulae}$$

$$S = \frac{1}{2} \text{ of the perimeter of the triangle}$$

$$= \frac{1}{2}(a + b + c) \quad A, b, c \text{ are the lengths of the sides of the triangle.}$$

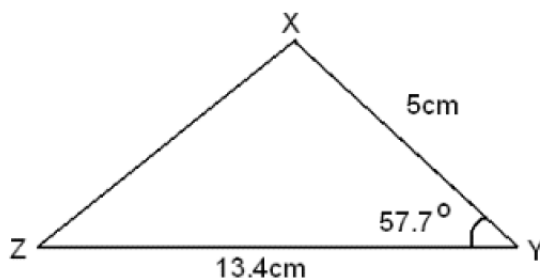
$$\begin{aligned} &= \frac{1}{2}(6 + 7 + 5) = 9 \quad \text{And } A = \sqrt{9(9-6)(9-7)(9-5)} \\ &= \sqrt{9 \times 3 \times 2 \times 4} \\ &= \sqrt{216} \\ &= 14.70 \text{ cm}^2 \end{aligned}$$

End of topic

ASSIGNMENT

Past KCSE Questions on the topic.

1. The sides of a triangle are in the ratio 3:5:6. If its perimeter is 56 cm, use the Hero's formula to find its area (4mks)
2. The figure below is a triangle XYZ. ZY = 13.4cm, XY = 5cm and angle XYZ = 57.7°



Calculate

- i.) Length XZ. (3mks)
 - i.) Angle XZY. (2 mks)
 - ii.) If a perpendicular is dropped from point X to cut ZY at M, Find the ratio MY: ZM. (3 mks)
- Find the area of triangle XYZ. (2 mks)

AREA OF QUADRILATERALS

Specific Objectives

By the end of the topic the learner should be able to:

- Find the area of a quadrilateral
- Find the area of other polygons (regular and irregular).

Content

- Area of quadrilaterals
- Area of other polygons (regular and irregular).

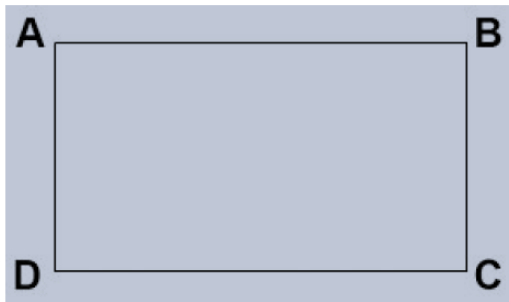
Introduction

Quadrilaterals.

They are four sided figures e.g. rectangle, square, rhombus, parallelogram, trapezium and kite.

Area of rectangle

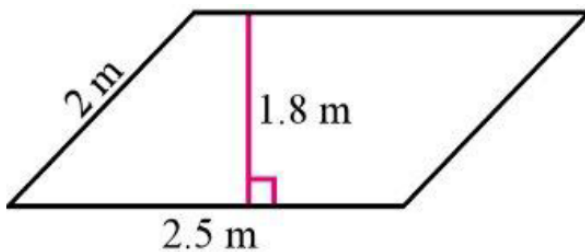
$$A = L \times W$$



AB and DC are the lengths while AD and BC are the width.

Area of parallelogram

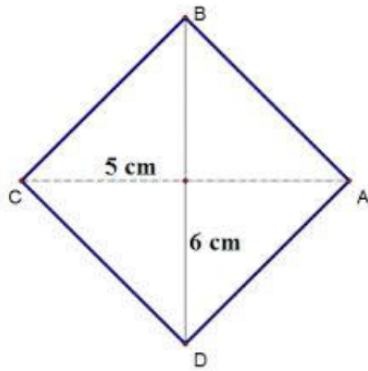
A figure whose opposite sides are equal and parallel.



$$\text{Area} = \text{base} \times \text{height} = 2.5 \times 1.8 = 4.5 \text{ cm}^2$$

Area of a Rhombus.

A figure with all sides equal and the diagonals bisect each other at 90° . In the figure below $BC = CD = DA = AB = 4$ cm while $AC = 10$ cm and $BD = 12$. Find the area



Solution

Find half of the diagonal which is $\frac{1}{2} \times 10 = 5$ cm

$$\text{Area of } \triangle BCD = \frac{1}{2} \times 12 \times 5 = 30 \text{ cm}^2$$

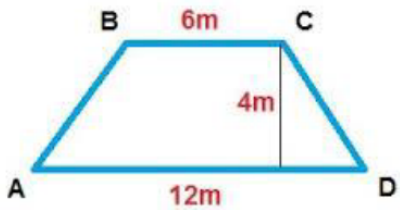
Area of $\triangle ABCD = 2 \times$ area of $\triangle BCD$

$$= 2 \times 30 \text{ cm}^2$$

$$= 60 \text{ cm}^2$$

Area of Trapezium

A quadrilateral with only two of its opposite sides being parallel. The area = $\left(\frac{a+b}{2}\right)h$



Example

Find the area of the above figure

Solution

$$\text{Area} = \left(\frac{6+12}{2}\right) 4$$

$$= 9 \times 4 = 36 \text{ cm}^2$$

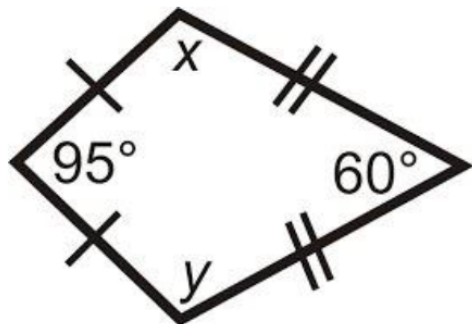
Note:

You can use the sine rule to get the height given the hypotenuse and an angle.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

Or use the acronym SOHCAHTOA

Rhombus



Example

In the figure above the lines marked // = 7 cm while / = 5 cm, find the area.

Solution

Join X to Y.

Find the area of the two triangles formed

$$\frac{1}{2} \times 5 \times 5 \times \sin 95^\circ = 12.45 \text{ (Triangle one)}$$

$$\frac{1}{2} \times 7 \times 7 \times \sin 60^\circ = 21.21 \text{ (Triangle two)}$$

Then add the area of the two triangles

$$12.45 + 21.21 = 33.67 \text{ cm}^2$$

Area of regular polygons

Any regular polygon can be divided into isosceles triangle by joining the vertices to the Centre.
The number of the polygon formed is equal to the number of sides of the polygon.



Example

If the radius is of a pentagon 6 cm find its area.

Solution

Divide the pentagon into five triangles each with 72° ie $(\frac{360}{5})$

$$\begin{aligned} \text{Area of one triangle will be} &= \frac{1}{2} \times 6 \times 6 \times \sin 72^\circ \\ &= 17.11 \end{aligned}$$

There are five triangles therefore

$$\text{AREA} = 5 \times 17.11$$

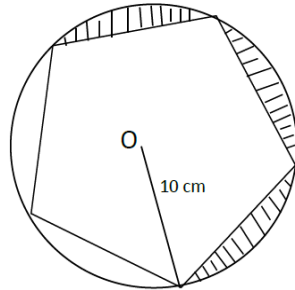
$$= 85.55 \text{ cm}^2$$

End of topic

ASSIGNMENT

Past KCSE Questions on the topic.

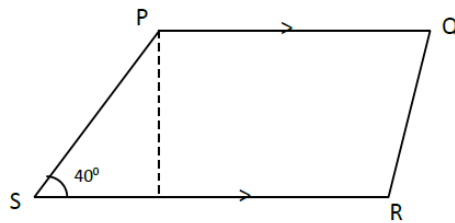
1.) The diagram below, not drawn to scale, is a regular pentagon circumscribed in a circle of radius 10 cm at centre O



Find

- (a) The side of the pentagon (2mks)
 (b) The area of the shaded region (3mks)

2.) PQRS is a trapezium in which PQ is parallel to SR, $PQ = 6\text{cm}$, $SR = 12\text{cm}$, $\angle PSR = 40^\circ$ and $PS = 10\text{cm}$. Calculate the area of the trapezium. (4mks)



3.) A regular octagon has an area of 101.8 cm^2 . calculate the length of one side of the octagon (4marks)

4.) Find the area of a regular polygon of length 10 cm and side n , given that the sum of interior angles of $n : n - 1$ is in the ratio 4 : 3.

1.) Calculate the area of the quadrilateral ABCD shown:-

